Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Engineering 57 (2013) 1230 – 1235

Procedia Engineering

www.elsevier.com/locate/procedia

11th International Conference on Modern Building Materials, Structures and Techniques, MBMST 2013

Development of Methods for Nonparametric Identification of Models of Mechanical Systems

Viktorija Volkova*

Department of Construction and Geomechanics, Faculty of Construction, SHEI National Mining University, Karl Marx av. 19, UA-49005 Dnepropetrovsk, Ukraine

Abstract

The analysis of existing methods of nonparametric identification of dynamic systems is shown in the article. The application of phase trajectories mappings on plane “acceleration – displacement” is suggested by author to nonparametric identification of mechanical systems models. The efficiency of the given method had estimated by it’s comparison with a known method of non-parametric identification.

Keywords: non-parametric identification, dynamic system, phase trajectories, phase plane “acceleration – displacement”, Chebyshev polynomials, time histories.

1. Introduction

There are a number of methods which are presently applied for the construction of the mathematical models of dynamic systems – the method of choice, the phenomenological approach and the method of identification.

The development of mathematical models for the real-world dynamic systems is substantially influenced by a judgmental factor. This is reflected in the degree of idealization chosen for the initial dynamic process, the opted model structure, and the selected type of non-linearity. The apparent advantage of the identification is that the process of model selection is based on the objective information obtained experimentally.

In spite of the fact that such models suffer a certain error, the results of their application represent, generally, a satisfactory approximation. For this reason the dynamic behaviour of a certain object can be described with several mathematical models differing from each other not only in the values of particular parameters, but also structurally.

2. The overview of the conventional methods of nonparametric identification

Both the static and dynamic processes in the mechanical systems may possess the non-linear elastic and/or dissipative characteristics, which must not be neglected. It is common knowledge that this is by no means always that a quite extensive and accurate description of the mechanical systems can be grounded merely on the basis of a direct express physical analysis. As a rule, some of the parameters have to be determined on the basis of experiments. In a number of instances even the structure of the mathematical model turns to be obscure [1–6, 15]. The identification of mechanical systems involves a great variety of tasks. These tasks differ from one another not only in the intended practical application of the mathematical model being constructed, but also in the amount of initial information.

* Corresponding author. Tel.: +380-50-561-5562; fax: +380-567-446-211.
E-mail address: drvev@mail.ru

doi:10.1016/j.proeng.2013.04.155

Most of the currently available methods of nonparametric identification are based on the polynomial approximation of models. Thus, the targets of research of the majority of the modern methods of nonparametric identification are the time histories (i.e. the time processes), more specifically, studying the recorded changes, which occurred in the displacement of points of the system under investigation, in the time domain. These methods are based on the application of the wavelet transform, the series of Wiener and Hamerstein. These approaches are cumbersome to implement and involve the use of the computer technology [2–9] and generate a need for storing significant amounts of initial information. The basic functions underlying these methods operate with the higher derivatives (of the fourth, fifth and sixth orders). The necessity to perform the multiple numerical differentiation of an original noise-distorted signal unavoidably leads to the building up of error accumulation and truncation, and that, in turn, significantly degrades the accuracy of a model. These methods are also highly restrictive in terms of the types of systems that can be identified and the modes of external testing excitation.

In view of the fact these methods are grounded on different approaches in their construction, it is difficult to draw a comparison between them and determine the optimal conditions for their effective applications. Generally, such methods require powerful computational resources and along with this demand intelligence of the user for their successful implementation and the correct interpretation of the results. This is the reason why now the applicability of the developed methods of identification is determined not only from the point of view of their scientific soundness, but also the efficiency in applications. It is apparent that the choice of a model structure involves some challenges, so, to solve the problems of nonparametric identification it is necessary to develop and apply new methods based on the investigations of dynamic behaviour characteristics of a system under study.

3. Sources of non-linearity

Non-linearity is a common feature shared by mechanical systems, whereas their linear behaviour is an exception. One of the primary sources of non-linearity in mechanical system dynamics is non-linearity of dissipative characteristics. In fact, energy dissipation is the least understood aspect in mechanical systems. Geometric non-linearity takes place when a structure undergoes large displacements and arises from the potential energy. Large deformations of flexible elastic beams, plates and shells are also responsible for geometric non-linearities Non-linearity may also result due to boundary conditions, for example, free surfaces in fluids, vibro-impacts due to loose joints or contacts with rigid constraints, clearances, or certain external non-linear body forces.

The structural systems involve large number of elements that are connected through the bolts and pins. Joints and fasteners are used to transfer loads from one structural element to another. Structural joints are regarded as major source of non-linear phenomena. The complex behaviour of connecting elements plays an important role in the dynamic characteristics, such as natural frequencies, mode shapes, and non-linear response characteristics to external excitations. The stresses and slip in the vicinity of contact regions determine the static strength, cyclic plasticity, frictional damping, and vibration levels associated with the structure. The need for developing methods for developing models of structures with joints has been discussed in papers [5, 8].

4. The phase trajectories and their representations in the nonparametric identification of models of mechanical systems

One of the trends in solving the direct problems of dynamics of structural elements is associated with the investigations into the behavior of the phase trajectories on the plane “displacement – velocity”. In spite of the fact that the geometrical approach has been successfully used for quite a long period of time, it is precisely this approach which forms the basis of nonlinear dynamics and provides possibilities to predict new effects in different fields of knowledge. from in its applications

The qualitative investigations of the dynamic system behavior are confined to studying the behaviour of trajectories in the phase space. Henri Poincaré laid the foundations of the qualitative theory for studying dynamic processes. A.A. Andronov [10, 11], E.A. Leontovich, I.I. Gordon and A.M. Lyapunov have made exceptionally valuable contributions to the development of the qualitative methods for investigation of dynamic systems.

The principal objective of the classical theory of qualitative research is to determine dynamic properties of systems without obtaining a closed-form analytical solution. Therefore, the phase trajectories on the planes $\left(y, \dot{y}\right)$ have been widely used for this purpose. Concurrently with the classical qualitative theory of differential equations, Poincaré in his fundamental works suggested the method of point mappings, which was later developed further in the works of N.V. Butenin, Y.I. Neymark, L.P. Shilnikov, and other scientists.

The sphere of application of these techniques has not been restricted to the autonomous oscillations. There are some other known works in which the trajectory have been implemented for solving an inverse problem of mechanics, that is identification. So, in research [12], the authors, due to the application of the Scheffer's graphic method, obtained the numerical assessments of the dissipative characteristics of the individual points of the phase space.

5. The phase trajectories in the expanded phase space

As an alternative to the above mentioned methods, a number of researchers in their works [13], [14] have suggested a relatively simple and direct approach to the identification of a wide class of mathematical models, Their approach is based on the application of the phase trajectories and their mappings in the extended phase space, every point of which is characterized by three coordinates – displacement, velocity, and acceleration. Particular concern in the phase trajectories and their representations on the phase planes "acceleration – displacement" is conditioned by the fact that they enable a more demonstrative interpretation of energy relationships in the system being investigated.

The monograph [14] sets off common and distinctive characteristics of the phase trajectories on the planes "acceleration – displacement" and "acceleration –velocity" that are inherent in the individual classes of non-linear dynamic models; the paper also presents the devised sets of classifiers and provides the description of interaction between the elements of the trajectories for the classification of the type of the model and (or) of the oscillation process.

6. Comparison of the methods of nonparametric identification

In order to assess the efficiency of the suggested methods for evaluation of the elastic and dissipative properties of mechanical systems [7], the results of the application of these techniques were compared to the experimental data obtained by a group of American scientists, whose research was sponsored with the grant from the National Science Foundation. The group included Professor S. F. Masri (Civil Engineering Department, the University of South California) and Professor T. K. Caughey (Division of Engineering and Applied Science, California Institute of Technology [7]. They have proposed a method of non-parametric identification, the fundamental idea of which is to use the Chebyshev polynomials in the approximation of a restoring force

$$T_n(\xi) = \cos\left(n \arccos(\xi)\right), \ -1 \le \xi \le 1. \tag{1}$$

In this particular case, $T_n(\xi)$ is the orthogonal Chebyshev polynomial of the first kind and $n\left(n = 0,\ 1,\ 2,\ \dots, m\right)$. The Chebyshev polynomials feature orthogonality

$$\int_{-1}^{1} \frac{T_n(\xi)\ T_m(\xi)}{\sqrt{1-\xi^2}} = \begin{cases} 0 & n \ne m, \\ \pi/2 & n = m \ne 0, \\ \pi & n = m = 0, \end{cases}$$

with respect to the weight $\omega(\xi) = \left(1-\xi^2\right)^{-1/2}$. Due to orthogonality of the polynomials, the calculation of the coefficients in the expression, approximating the process, becomes much easier than for non-orthogonal polynomials. The coefficients of the approximating polynomial equation do not depend on the order of the initial polynomial equation. Another important characteristic of the Chebyshev polynomials is that they have nearly equal errors. It lies in the fact that the error of approximation varies within the range of measurements between two practically equal limits.

As an example, in the work [7], the authors considered the system with one degree of freedom,

$$m\ddot{y} + f(y, \dot{y}) = P(t), \tag{2}$$

wherein the restoring force $f(y, \dot{y})$ is described by the following equation:

$$f(y,\ \dot{y}) = m\left[2\zeta\omega\dot{y} + \omega^2\left(1 + \ \beta y^3\right)\right], \tag{3}$$

where $y,\ \dot{y}$ are the generalized displacement and the velocity. The parameters in the Eq. (3) have taken the following values: $m = 1$; $\zeta = 0,05$; $\omega = 1$; $\beta\ = 0,003$.

The mechanical system under study was subjected to the external excitation of the type:

$$P(t) = P_1 \sin\left[\Omega(t)t\right],$$

$$\Omega(t) = \Omega(0) + \frac{\Omega(T_s) - \Omega(0)}{T_s} \tag{4}$$

where $P_1 = 4$ is the amplitude of the external excitation, and $\Omega(t)$ is the frequency variable of the external excitation linearly varied in the time interval $[0,\ T_s]$.

The main purpose of research [7] was the estimation of the parameters of the restoring force $f(y,\ \dot{y})$, which has been approximated by the function $\tilde{f}(y,\ \dot{y})$ of the type:

$$f(y,\dot{y}) = \tilde{f}(y,\dot{y}) = \sum_{i=0}^{n1-1} \sum_{j=o}^{n2-1} C_{ij}\, T_i(y) T_j(\dot{y}), \tag{5}$$

By means of numerical simulations during the research, the authors have obtained the values of displacements, velocities and accelerations and formed the corresponding vectors

$$y_k = y(t_k);\ \dot{y}_k = \dot{y}(t_k);\ \dot{y}_k = \dot{y}(t_k),\ k = 1,\ 2, \ldots, n, \tag{6}$$

where

$$t_k = (k-1)\Delta t \quad \text{and} \quad n = \frac{T_s}{\Delta t} + 1. \tag{7}$$

In view of the fact that the work [7] is lacking the data on the values of the initial conditions, while comparing the results, it was assumed that the displacement and the velocity of the system prior to displacement were equal to zero, i.e., in the initial moment of time the system was in equilibrium.

Due to the absence of cross terms, the restoring force can be expressed as the sum of two independent functions, one of which depends only on the displacement, and the second one on the velocities.Thus, taking into account that certain moments of time satisfy the relationships

$$f(y,0) = r(y) = m\omega^2 \left(1 + \beta y^3\right);$$

$$f(0,\ \dot{y}) = h(y) = 2m\zeta\omega\dot{y}, \tag{8}$$

the restoring force can be expressed as the sum of

$$f(y,\ \dot{y}) = r(y) + h(\dot{y}). \tag{9}$$

In a generalized case, the surface of the restoring force in the space $(y, \dot{y}, \ddot{y})$, can be specified by the approximated polynomial

$$f(y,\dot{y}) = \tilde{f}(y,\dot{y}) = \sum_{i=o}^{n1-1} a_i T_i(y) + \sum_{j=0}^{n2-1} b_j T_j(\dot{y}), \tag{10}$$

where a_i are the coefficients of the least- squares polynomial approximation obtained from the experimental data, which satisfy the condition $|\dot{y}| \approx 0$; and where b_i are the coefficients obtained for the points, which satisfy the same condition $|y| \approx 0$.

A part of the time process that corresponds to the oscillations with the resonance amplitudes has been obtained on the basis of the results of numerical modeling.

Processing the recorded series of displacements, velocities and accelerations at the constant time interval of $T/\Delta t = 20$ resulted in three vectors, each comprising 981 elements in length. The independent variables $y, \dot{y}$ in the Eq. (6) need to be subjected to transformation in such a manner that they meet the requirements imposed to the area of changes of the argument $-1 \le \xi \le 1$ in the Chebyshev polynomials. After the transformation of values of the displacements and velocities, two matrix were formed, the elements of which satisfy the conditions $|\tilde{y}| \le 0,05$; $|\dot{\tilde{y}}| \le 0,05$, respectively:

In addition, provided the data distribution is uniform, the approximation of the function is performed according to the points corresponding to the roots of the Chebyshev polynomials.

$$\xi_i = \cos\left[(2i+1)\pi / 2n\right],\ i = 0,\ 1,\ 2,\dots,n-1 . \tag{11}$$

The roots of the Chebyshev polynomials are arranged symmetrically with respect to the origin of coordinates in the interval [–1; 1] and unevenly with respect to the ends of the segment; that is the closer the roots are to the ends of the segment, the tighter they are to each other. Thus, the application of the polynomials as the approximating functions provides a close agreement between the tabulated input function and the approximating functions in the extreme points of the interval of changes of the argument. However, the question about the quality in the middle of the interval of the argument variations remains open. Thus, for those functions, which have the interruptions or feature any peculiarities in the middle of the interval of changes of the argument, it is necessary to artificially increase the order of the polynomial.

One of the shortcomings of the above described procedure in the work [7] is that the criteria proposed by the authors for selection of elements from the original data set do not allow formulating a satisfactory approximating expression. So, in the beginning of the motion the values of displacements and velocities are close to the values and insignificant $\tilde{y} \approx \dot{\tilde{y}} \le 0,005$, and the velocity and the displacement have the same phase (see Table 1). Therefore, one can conclude that the use of the expression (8) in order to discriminate between the elastic and dissipative components of the restoring force in this interval of the time process is not appropriate. This disadvantage can be overcome either by setting the relevant initial conditions or excluding the initial interval from the time process analysis.

The cited paper does not explain the reasoning of choice of the type of the external testing excitation. It should be noted that chosen type of excitation does allow performing the efficient assessment of the resonance values of the displacements, velocities and accelerations in the system being studied. However, this choice has its drawbacks. For instance, in one interval of the external excitation, at the beginning of its application and at its end, the number of points is not equal; for this reason, the response of the system cannot be assessed correctly.

Table 1. The results of numerical simulations $\Delta t = T/20$

k	$\tilde{y}$	$\dot{\tilde{y}}$	$f(\tilde{y},\dot{\tilde{y}})$
1	0,012003	0,006084	0,012271
2	0,01225	0,009511	0,088237
3	0,013936	0,014745	0,279775
4	0,018318	0,021125	0,624699
5	0,026317	0,027823	1,143013
6	0,038391	0,033916	1,836156
7	0,054453	0,038462	2,687784
8	0,073841	0,040578	3,662581
9	0,095324	0,039528	4,700154
10	0,117167	0,034847	5,7061
11	0,15332	0,026491	6,54991
12	0,163214	0,014958	7,082631
13	0,165281	0,001298	7,177883
14	0,15864	–0,013054	6,779385
15	0,143332	–0,026596	5,924502
16	0,120259	–0,038085	4,725222
17	0,090958	–0,046722	3,318555
18	0,057315	–0,052143	1,818295
19	0,021347	–0,05427	0,293338
20	–0,014926	–0,053145	–1,223364

Moreover, the condition $|\tilde{y}| \le 0,05$; $|\dot{\tilde{y}}| \le 0,05$ is not strictly satisfied; and consequently, the points that satisfy these conditions, do not lie at the equal distances from one another. Owing to this, the coefficients a_i and b_i will change their values depending on the length of recordings of the applied external excitation and its frequency.

It is apparent that the Gram-Schmidt method is a more rational technique for the estimation of the coefficient values a_i and b_i of the Chebyshev polynomials, because this method is applicable to the cases of uneven data distribution.

By means of processing of the obtained values of the displacements and velocities, the following values were calculated for the coefficients of the Chebyshev polynomials: $a_0 = 0,23$; $a_2 = 0,16$; $a_3 = 51,4$; $b_0 = 0,24$; $b_1 = 3,54$.

Using the conventional expressions for the Chebyshev polynomials, by way of substituting the values of the coefficients, and reducing the similar terms in the approximating equation of the restoring force, the expression has been brought to the following form:

$$\tilde{f}(y,\ \dot{y}) = \tilde{r}(y) + \tilde{h}(\dot{\tilde{y}}) = 0,927y + 1,65\cdot 10^{-4}y^2 + 0,0024y^3 + 0,28 + 0,046\dot{y}. \tag{12}$$

As it can be seen from Eq. (12), the error of the coefficient estimates increases depending on the order of members. So, the error in the estimation of the parameter ω^2 is 7.3%, and that of the coefficient β is 20%. It should also be noted the emergence of quadratic and free members in the approximating equation; that this points to the non-symmetry of the characteristics of the restoring force, but it contradicts to the original model.

Under such circumstances, the most adequate, quite simple method that can be employed for the non-parametric identification is the one applicable for a wide class of mechanical systems, which have one degree of freedom and manifest the nonlinear properties inherent in the real-world systems. The method is based on the use of information pertaining to the displacement and acceleration and taking into account the external excitation acting upon the system. Anticipatory neglecting the influence of the dissipation, it is assumed that the characteristics of the elastic force can be determined by the following equation:

$$r\ (y_k) = c\ -\ m_1\ddot{y}_k\ . \tag{13}$$

The construction of the mappings of the phase trajectories on the plane $(y,\ \ddot{y})$ is close to the method of processing the time processes for peaks. The assessment of the values of the velocity and displacement was carried out in discrete moments of time that meet the condition $c = F(t_o) = F(t_k) = 0$. The point mappings enabled the construction of the polynomial trend. In order to assess the statistical reliability of the obtained polynomial trend, the value of the multiple coefficient of determination has been determined; the value $R^2 = 0,982$ of this coefficient exceeded the standard value $\tilde{R}^2 = 0,6$. The deviation of the value of the coefficient of the polynomial trend from the actual value amounted to 7%, and for the frequency of self-oscillations it was ω^2 – 5%.

References

[1] Grop. 1979. Identification of mechanical systems. Moscow: Mir. 312 p.
[2] Adams, D. E., Allemang, R. J., 1998. "Survey of nonlinear detection and identification techniques for experimental vibrations", Procc. of ISMA 23, pp. 269-280.
[3] Kononenko, V. O., Plakhtiyenko, M. P., 1976. Methods of Identification of Mechanical Oscillating Systems. Kiew: Naukowa dumka. 114 p.
[4] Plakhtiyenko, M. P., 2000. Methods of Identification of Mechanical Oscillating Systems, International Applied Mechanics 36(12), pp. 38-68.
[5] Ibrahim, R. A., Pettit, C. L., 2005. Uncertainties and dynamic problems of bolted joints and other fasteners, Journal of Sound and Vibration 279, pp. 857–936.
[6] Kulisiewics, M., 2005. Modelling and identification of non-linear mechanical systems under complex load. Wroclaw: Oficyna wydawnicza Politechniki Wroclawickiej, 195p.
[7] Masri, S. F., Caughey, T. K., 1979. A Nonparametric Identification Technique or Non-linear Dynamic Problems, Transactions of ASME, Journal of Applied Mechechanics 46, pp. 433–447.
[8] Crawley, E. F., 1987. Force-state mapping identification of nonlinear joints, Institute of Aeronautics and Astronautics Journal 25, pp. 1003–1010.
[9] Kerschen, G., Worden, K., Vakakis, A.F., Golinval, J.-C., 2006. Past, present and future of nonlinear system identification in structural Dynamics, Mechanical Systems. Signal Process 20(3), pp. 505–592.
[10] Andronov, A. A., Leontovich, E. A., Gordon, I. I., Mayer, A. G., 1966. The Qualitative Theory of Dynamic Systems of the Second Order. Moscow: State publishing house of the physical and mathematical literature. 568 p.
[11] Andronov, A. A., Leontovich, E. A., Gordon, I. I., Mayer, A. G., 1973. Qualitative theory of second order systems. New-York e.t.c.: John Wiley & Son. 524 p.
[12] Forshing, H., Manea, V., 1973. Zur analytischen Behandlung des nichtlinearen aroelastischen Galloping Problems, Ing. Archiv Bd. 42, pp. 178-193.
[13] Volkova, V. E., Schneider, K., 2005. Qualitative theory and identification of dynamic system with one degree of freedom, International Applied Mechanics 41(6), pp. 134-139.
[14] Volkova, V. E., Kazakevitch, M. I., 2009. Application of the extended phase trajectories to identification of chaotic systems, Inverse Problems in Science and Engineering 17(2), pp. 203–212. ISSN 1741-5977
[15] Worden, K., Tomlinson, G. R., 2001. Nonlinearity in Structural Dynamics: Detection, Identification and Modelling. New-York: Institute of Physics Publications. 678 p.